INMO 2017: Practice Test 2

Time : 4 hours **December 17, 2016**

Instructions:

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- Answer all the questions. Draw neat Geometry diagrams.
- Each question is worth 17 marks. Total marks: 102.
- Answer to each question should start on a new page. Clearly indicate the question number.
- Mathematical reasoning will be taken into consideration while assessing the answers.

Questions:

1. Inside any convex 2017-gon, determine the maximum number of distinct diagonals that can be constructed such that no two intersect each other.

2. Find the number of integers n such that:

$$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{5} \right\rfloor = n$$

where $\lfloor \ \rfloor$ denotes the greatest integer function.

3. Let $P(x)$ and $Q(x)$ be polynomials with real coefficients. Given that for all $x \in \mathbb{R}$, we have $P(x) \neq Q(x)$, and $P(Q(x)) = Q(P(x))$; prove that for all $\forall x \in \mathbb{R}: \ P(P(x)) \neq Q(Q(x))$.

4. Find all $n \in \mathbb{N}$ such that for every positive divisor d that divides n, we have $(d+1)$ divides $(n+1)$.

5. If a, b, c are non-negative real numbers such that $a + b + c = 1$, then prove that:

$$\sum_{cyclic} \sqrt{a + \frac{(b-c)^2}{4}} \leq 3\sqrt{3} - 2 \sum_{cyclic} \sqrt{a}$$

6. On the circumcircle Γ of an acute ΔABC, let L be the midpoint of minor arc BC. Let T be any point on minor arc BL. Let Γ_2 be a circle that is externally tangent to Γ at T, and to side AB extended at a point X. Let XT intersect Γ a second time at point S, and let CY be the tangent segment from C to Γ_2, such that Y lies on Γ_2, and X, Y lie on opposite sides of line AL. Prove that the lines AL, SC and XY are concurrent.