

INMO 2024 Practice Test

Q1 Given a graph with 50 vertices and 40 edges such that each vertex has degree atleast 1, prove that there exists a subset of 10 edges that don't share any vertex. Does there always exist a subset of 11 edges with a similar property?

Q2 Given a fixed circle Γ and a fixed chord PQ inside it, let A be the midpoint of minor arc PQ . Let X be a variable point on seg PQ ; and let the line AX meet Γ a second time at S . Let the perpendicular line to AX at X meet Γ at point T such that P, T are on opposite sides of the line AX . Find the locus of the midpoint of chord ST as X varies on seg PQ .

Q3 Find all $k \in \mathbb{N}$ such that the polynomial $x^{2k+1} + x + 1$ is divisible by the polynomial $x^k + x + 1$. For each such k , find all $n \in \mathbb{N}$ such that $x^n + x + 1$ is divisible by $x^k + x + 1$.

Q4 Find the smallest possible $n \in \mathbb{N}$ with the following property: From any n distinct rectangular carpets all of whose sides are positive integers less than 10, there exist carpets A, B, C such that A can cover B and B can cover C .

Q5 Prove that for any prime p , there exist an infinite number of values $m \in \mathbb{N}$ such that p is a divisor of $2^{m+1} + 3^m - 17$.

Q6 Let ABC be a triangle with $AB > AC$; let O be its circumcentre and Γ be its circumcircle. Let T be the diametrically opposite point to A in Γ . Let the tangent to Γ at T meet line BC at D . Let line OD meet sides AB, AC at points P, Q respectively. Prove that $PO = OQ$.