

PRMO 2018: Practice paper II

Maharashtra and Goa Region

Time: 2.5 hours

03 June 2018

Instructions

- The answer to each question may be an integer or a fraction. Students should write their numerical answer in the provided field on the answer sheet.
- No other markings should be made on the answer sheet. Blank sheets will be provided for rough work.
- The use of calculators of any kind is not allowed.

Questions

- Q1 Let a, b, c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. What is the value of $a^2 + b^2 - c^2$?
- Q2 A subset B of the set of first 100 positive integers has the property that no two elements of B sum to 125. What is the maximum possible number of elements in B ?
- Q3 O and I are the circumcentre and incentre of $\triangle ABC$ respectively. Suppose O lies in the interior of $\triangle ABC$ and I, B, O, C are concyclic. What is the magnitude of $\angle BAC$ in degrees?
- Q4 Let $S_n = n^2 + 20n + 12$, where n is a positive integer. What is the sum of all the possible values of n for which S_n is a perfect square?
- Q5 What is the smallest possible natural number n for which the equation $x^2 - nx + 2014 = 0$ has integer roots?
- Q6 Let XOY be a triangle with $\angle XOY = 90^\circ$. Let M and N be the midpoints of the legs OX and OY respectively. Suppose that $XN = 19$ and $YM = 22$. What is XY ?
- Q7 Let $A_1A_2 \cdots A_{12}$ be a convex polygon. Let N be the number of ways in which one can color its vertices using red and blue, so that no three consecutive vertices have the same color. Find the sum of the digits of N .
- Q8 In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 17. What is the greatest possible perimeter of the triangle?
- Q9 ABCD is a square of side length 1. Equilateral triangles AYB and CXD are drawn such that X and Y are inside the square. What is the length of XY ?

- Q10 In how many ways can a 6×6 floor be tiled using 6 tiles of size 2×3 ? Tiles can be placed horizontally or vertically, and no two tiles should overlap.
- Q11 Let $S = \{1, 2, 3, 4, 5, 6\}$. What is the number of ordered pairs (A, B) such that A is a subset of B ; and B is a subset of S ?
- Q12 If a, b, c are natural numbers such that b is the arithmetic mean of a and c , and b is also the geometric mean of $a + 1$ and $c - 1$, then find the value of $(a - b)^2 + (b - c)^2 + (c - a)^2$.
- Q13 Find the number of ordered pairs (x, y) which satisfy the equation $x^2 + 5 = y^2 + x$, where x, y are integers.
- Q14 If all the 3-digit numbers that can be formed using the digits 1, 2, 3, 4 (repetitions allowed) are written in ascending order, then find the sum of the digits of the 100th number in the list.
- Q15 In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on line segment CD , so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E . What is the area of $\triangle AEB$?
- Q16 A student writes a 2-digit number N on the board. Then he keeps repeating the following operation: multiply the number on the board by 2, add 1 to the result, and write this new number on the board (replacing the previous written number). After some time, when the teacher comes, she finds the number 165 written on the board. Then find the value of N .
- Q17 Let $P(x)$ be a cubic polynomial such that $P(-1) = 1$ and $P(1) = -1$. If the coefficient of x^3 is 1; find the sum of the roots of the polynomial.
- Q18 $ABCD$ is an isosceles trapezium such that the lengths of each diagonal is 7, and the parallel sides are of length 4 and 6 respectively. Find the perimeter of the trapezium.
- Q19 A coffee mug is cylindrical in shape, with height 10 and radius 7. If it costs 1 rupee per square unit to paint the mug; find the total cost to paint it from all sides (both inside and outside, including the base).
- Q20 Person A visits the beach once in every 4 days, while B visits the beach once in every 6 days. Find the maximum number of times that they will simultaneously visit the beach in the month of May.
- Q21 If $3^x + 2^y = 985$ and $3^x - 2^y = 473$, what is the value of xy ?
- Q22 Let x, y, z be positive integers such that $xy + yz = 2xz$. If $\sqrt[x]{10} = \sqrt[y]{20} = \sqrt[z]{N}$, then find the value of N .
- Q23 Find the number of ordered pairs (p, q) of prime numbers such that $p^2 + 7pq + q^2$ is a perfect square.
- Q24 Find the least possible value of $a + b$ where a, b are positive integers such that 11 divides $a + 13b$ and 13 divides $a + 11b$.

- Q25 Line l passes through a point A . Two circles with centers P and Q are of radii 3, 5 respectively. Both circles are tangent to l at point A ; and both of them lie on the same side of line l . If a point B lies on l such that $PB = 10$, then find the area of $\triangle PQB$.
- Q26 Find the number of ordered pairs (m, n) of positive integers such that the product of their GCD and LCM is 100, while the ratio of LCM/GCD is 10.
- Q27 Consider the expression $f(n) = n^{99} + 2n^{98} + 3n^{97} + \dots + 98n^2 + 99n + 100$. What is the remainder of $f(2)$ on dividing by 100?
- Q28 Four workers A, B, C, D are working to finish a project over the course of 6 days. On each day, exactly two of them are working, while the others are resting. In how many days would they have finished the project if all of them worked all the time?
- Q29 x, y, z are real numbers such that $x + y + 2z = 6$ and $x^2 + y^2 + 4z^2 = 12$. Find the value of $x^3 + y^3 + z^3$.
- Q30 A company makes chocolates that are shaped like a small pyramid. The base of the pyramid is a square of size 3cm, and each of the four sloping sides is an equilateral triangle of side length 3cm. Find the volume of the chocolate.

