

Regional Mathematical Olympiad - 2016: Practice Paper

Time : 3 hours

September 08, 2016

Instructions:

- Calculators (in any form) and protractors are not allowed.
 - Rulers and compasses are allowed.
 - Answer all the questions. Draw neat Geometry diagrams.
 - Marks for each question are mentioned next to the same. Maximum total marks: 100.
 - Answer to each question should start on a new page. Clearly indicate the question number.
 - Mathematical reasoning will be taken into consideration while assessing the answers.
1. (16 marks) Find all positive integers m, n such that $2^m + 2016 = 2^n$.
 2. (16 marks) Given $\triangle ABC$, let P and Q be the feet of the perpendiculars drawn from point A onto the internal bisectors of angles B and C respectively. Prove that PQ is parallel to BC .
 3. (16 marks) Find all real numbers x, y, z that satisfy the following system of equations:

$$6x - 5y = xy$$

$$6y - 5z = yz$$

$$6z - 5x = zx$$

4. (16 marks) There is a circular table with 18 identical chairs around it. Find the number of ways in which 13 students and 5 teachers can be seated, so that no 4 students are seated in consecutive chairs. (Note that any two seating arrangements which are identical by rotation, will be considered identical)
5. (16 marks) Given a fixed segment BC and a line l parallel to it, find with proof the position of a point A on line l , for which the measure of $\angle BAC$ is the maximum.
6. (20 marks) Find the number of integers m such that **exactly one** of the roots of the following quadratic equation is an integer:

$$2x^2 - mx + 125 = 0$$