Pre-RMO Geometry test

10 questions · 100 marks · 2 hours

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July 8, 2017

Instructions

- The answer of each question is in the form of a number in the range 00 to 99. You are expected to darken the corresponding digits bubble in the answer sheet, for each question. (The upper row of $0-9$ digits is the tens' place, and the lower row is the units' place.)
- There are 10 questions, each worth 10 marks. There is no negative marking. Maximum marks are 100.
- You can do rough work in your notebook.

Questions

1. In ΔABC, let AD be the internal bisector of $\angle A$, with D lying on side BC. If the areas of ΔABD and ΔADC are 20 and 25 square units respectively, and $AB + AC = 27$, then find $\angle A$ in degrees.
2. Circles O_1 and O_2 intersect in points A and B, such that the distances of their centers C_1 and C_2 from the line AB are 4 and 9 respectively. If $AC_1 \perp AC_2$, then find the length of seg AB.
3. In ΔABC, let H, O be the orthocenter and circumcenter respectively. If B, H, O, C are concyclic, and the lengths of sides AB, AC are 5 and 7 respectively, then find the value of BC^2.
4. Let l_1, l_2, l_3 be three parallel lines. Let l_4 be a transversal line that cuts l_1, l_2, l_3 at points A, B, C, in that order. Similarly, let l_5 be another transversal, that cuts l_1, l_2, l_3 at D, E, F, in that order. Given that the areas of $ABED$ and $BCFE$ is equal, and that $AD = 63$, and $CF = 112$, then find BE.
5. Given quadrilateral $ABCD$, whose diagonals intersect in point P. Given the lengths $AB = 5$, $BC = 8$, $CD = 13$, $DA = 9$, $BP = 3$, $PD = 4$, find the value of $AP^2 + PC^2$.
6. Find the area of ΔABC, given the ex-radius $r_1 = 20$, the inradius $r = 4$, and $\angle A = 90°$.
7. x is a real number. Find the number of values of x in the range 0 to 1000, for which $\sin(x°) = \frac{1}{2}$, where the unit of x is assumed to be degrees.
8. Find the number of values of x that satisfy the equation $\sin(x) + \cos(x) = 2$.
9. In ΔABC, let H be the orthocenter, and X, Y be the midpoints of segments AH and BC. If $AX = 12$ and $BY = 16$, then find the length of segment XY.
10. Consider an 8-sided polygon, inside which we draw all the possible diagonals, by joining all the vertices to each other. Find the number of points of intersection of these diagonals (assume that no three diagonals are concurrent).

Bonus question and today's puzzle! Find the formula for Q10, for any N-sided polygon. In addition, also count the number of regions that are formed inside the polygon. (For eg, with $N = 5$, we get 5 points of intersection, and 11 regions)